1 stepped pressure equilibrium code : sw02aa

Contents

1 stepped pressure equilibrium code : sw02aa **1**

1.1 outline 1

1.2 spectral width 1

1.3 tangential variations 1

1.4 extremizing condition 1

1.5 comments 2

1.1 outline

1. Compute spectral-width extremizing condition.

1.2 spectral width

1. The geometry of an interface is described by two functions, $R = \sum_j R_j \cos(m_j\theta - n_j\zeta)$ and $Z = \sum_j Z_j \sin(m_j\theta - n_j\zeta)$. (See `global` and `co01a` for more details.)

2. The spectral width is defined

$$M = \frac{1}{2}\sum_j (m_j^p + n_j^q)\left(R_j^2 + Z_j^2\right). \tag{1}$$

   where $p \equiv$ `pwidth`, $q \equiv$ `qwidth` are positive integers given on input, and $m_j^p = 0$ for $m_j = 0$, $n_j^q = 0$ for $n_j = 0$.

1.3 tangential variations

1. We seek to extremize the spectral width without changing the geometry of the interface. Accordingly, we restrict attention to tangential variations, i.e. variations of the form

$$\delta R = R_\theta\,\delta u, \tag{2}$$

$$\delta Z = Z_\theta\,\delta u. \tag{3}$$

2. To preserve stellarator symmetry, we consider $\delta u = \sum_k u_k \sin(m_k\theta - n_k\zeta)$.

3. The variations in the Fourier harmonics of R and Z are given by

$$\delta R_j = \oint\!\!\oint d\theta d\zeta\;\; R_\theta\,\delta u\;\cos(m_j\theta - n_j\zeta), \tag{4}$$

$$\delta Z_j = \oint\!\!\oint d\theta d\zeta\;\; Z_\theta\,\delta u\;\sin(m_j\theta - n_j\zeta), \tag{5}$$

4. The first variation in M as

$$\delta M = \oint\!\!\oint d\theta d\zeta\;\;(R_\theta\,X + Z_\theta\,Y)\,\delta u, \tag{6}$$

   where $X = \sum_j (m_j^p + n_j^q) R_j \cos(m_j\theta - n_j\zeta)$ and $Y = \sum_j (m_j^p + n_j^q) Z_j \sin(m_j\theta - n_j\zeta)$

1.4 extremizing condition

1. The condition that $\delta M = 0$ for arbitrary δu is

$$I \equiv R_\theta\,X + Z_\theta\,Y = 0. \tag{7}$$

2. The derivatives of M with respect to the u_k are given

$$\frac{\partial M}{\partial u_k} = \oint\!\!\oint d\theta d\zeta\;\;(R_\theta X + Z_\theta Y)\sin(m_k\theta - n_k\zeta). \tag{8}$$

   These quantities are provided by an fast Fourier transform of I, which is computed in `cb02aa`.

1.5 comments

1. For `pwidth`$=2$, and ignoring the n^q term, we see [1] that $X \equiv -R_{\theta\theta}$ and $Y \equiv -Z_{\theta\theta}$, and the extremizing condition reduces to $R_\theta R_{\theta\theta} + Z_\theta Z_{\theta\theta} = 0$, which is equivalent to the equal arc length condition, $R_\theta^2 + Z_\theta^2 = const$.

sw02aa.h last modified on 2012-12-18 ;

[1] S. P. Hirshman and J. Breslau. Explicit spectrally optimized fourier series for nested magnetic surfaces. *Phys. Plasmas*, 5(7), 1998.